TECH TIP # 37

HARDI

HEATING, AIRCONDITIONING & REFRIGERATION DISTRIBUTORS INTERNATIONAL

One of a series of dealer contractor technical advisories prepared by HARDI wholesalers as a customer service.

SOLVING SERIES/PARALLEL CIRCUITS

Please study this Tech Tip along with assignment 4 in Basic Electricity.

Parallel circuits differ from series circuits in that the current divides into a number of separate, independent branches. In other words, if the current has more than one path to flow through the circuit, it is a parallel circuit. Moreover, these branches may have different resistances, and therefore, the value of current in each branch may be different. The loads that are added in parallel result in additional paths for current flow. This, in effect, decreases resistance. In the same manner, the more outlets a water pipe has, the smaller the total resistance and the greater the water flow.

There are electrical rules or laws for series and parallel circuits. These should be committed to memory.

THREE LAWS --- SERIES CIRCUITS

LAW # 1 --- THE SAME CURRENT FLOWS THROUGH ALL PARTS OF THE CIRCUIT

When working with series circuits, the current through each individual component is the same through every other part of the circuit. This means if you had three 15 ohm resistors wired in series, the current flow through the first one would be the same through the second and the third. Knowing the current value of one component would help you determine the other unknown values.

LAW #2 -- RESISTANCES ARE ADDITIVE

The formula for resistors wired in series is $R_T = R_1 + R_2 + R_3…..$
Therefore, to find the total equivalent resistance of three 15 ohm resistors wired in series, just add them up. $R_T = 15 + 15 + 15 = 45$ ohms.

LAW #3 -- VOLTAGE DROP ACROSS EACH RESISTANCE EQUALS APPLIED VOLTAGE

If you had three equal resistors wired in series with an applied voltage of 120 volts, each resistor would have a voltage drop of 40 volts. $Voltage_{Total} = 40 + 40 + 40 = 120$ volts.

THREE LAWS --- PARALLEL CIRCUITS

LAW #1 --- THE SAME VOLTAGE DROP IS ACROSS EACH ELEMENT

This law is perhaps the most abused of the electrical laws. The fallacious concept that prevails in the mind of many is that differing resistances result in differing voltage drops in a branch circuit. This is completely in error, and Figure 15 corroborates this. Four parallel branches or elements are connected to a 24 volt battery. Although all branches have different resistors, they are connected across the same voltage source. The battery potential between the minus and positive electrodes is 24 volts. Each branch, in effect, is an independent connection from the minus electrode to the positive electrode. Therefore, regardless of branch resistance, each branch or element must have a 24 volt drop.

LAW #2 --- THE TOTAL CURRENT IS EQUAL TO THE SUM OF THE CURRENTS FLOWING IN THE INDIVIDUAL BRANCHES

If a river divides into a number of branches, and later they rejoin, no change in water volume occurs. The initial volume of water merely divides according to the resistance offered by the various branches. When the branches rejoin into one stream, the leaving volume of water is the same as the starting volume. The sum of the branch volumes, however, would have to equal the total volume. There is no point where water could be gained or lost. A parallel electrical circuit is similar. In Figure 15, current flow in branch #1 would be the 24 volts impressed across the branch divided by the branch resistance of 3 ohms or

$$I = \frac{E_1}{R_1} = \frac{24 \text{ volts}}{3 \text{ ohms}} = 8 \text{ amperes}$$

In branch #2, it would be: $\dfrac{24 \text{ volts}}{4 \text{ ohms}} = 6 \text{ amperes}$

In branch #3, it would be: $\dfrac{24 \text{ volts}}{6 \text{ ohms}} = 4 \text{ amperes}$

In branch #4, it would be: $\dfrac{24 \text{ volts}}{4 \text{ ohms}} = 6 \text{ amperes}$

Grand total $= 8 + 6 + 4 + 6 = 24$ amperes

Therefore the sum of the branches would be 24 amperes. 24 amperes would originate at the negative terminal and flow until it divided at the branches. After the branches rejoin into one conductor the branch currents combine again into a 24 ampere flow.

LAW #3 --- TOTAL RESISTANCE IN A PARALLEL CIRCUIT IS EQUAL TO THE APPLIED VOLTAGE DIVIDED BY THE TOTAL CURRENT

Hence, the effective or total resistance in Figure 15 would be:

$$\frac{E_T}{I_T} \text{ or } \frac{24 \text{ volts}}{24 \text{ amperes}} = 1 \text{ ohm}$$

It may seem unusual for a circuit with branch resistances of 3 ohms, 4 ohms, 6 ohms and 4 ohms to combine into a total resistance of only 1 ohm. The 1 ohm is less than the resistance of the smallest branch resistor. This is logical when it is realized that each branch represents another outlet for electrical energy or in the case of piping for water flow. Each outlet contributes to a lessening resistance. In effect, the multiple outlets are equivalent to one large conductor or water pipe with their resultant decreases in resistance.
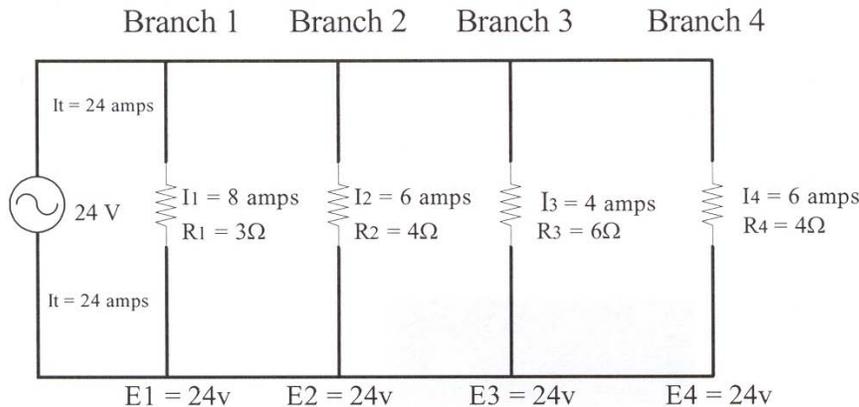

Figure 15
Study Diagram 15

Mini Quiz #15 (see answers on last pages)

If the above circuit has the values of R_1 = 12 ohms, R_2 = 8 ohms, R_3 = 4 ohms and R_4 = 24 ohms, then the total resistance would equal. Underline the correct answer.

a. 2 ohms   b. 4 ohms
c. 3 ohms   d. 1.5 ohms

SHORTCUT METHODS FOR COMBINING PARALLEL RESISTANCES

Three rules also apply to the shortcut methods for computing total branch resistances.

RULE #1

If all branch resistors are identical, the total resistance is simply the resistance of one branch divided by the number of branches, e.g., five 10 ohm resistors would be:

10 ohms/5 branches = 2 ohms total resistance

Rule #1 is illustrated in Figure 16. Computing resistance by the long method necessitates the determination of total current which is 30 amperes. Then dividing 60 volts by 30 amps yields an effective resistance of 2 ohms. In contrast merely taking the value of one resistor, 10 ohms, and dividing this by the number of branches 5, yields the same answer -- 2 ohms. Moreover, the answer is instantaneously computed.

RULE #2

This is called the product over sum method. Here the product of each pair of resistances is divided by the sum of the two resistances. The long method and the product over sum method are illustrated by Figure 17.

In the conventional method, current is computed for each branch. Then current is totaled as a value of 12 amperes. 24 volts are divided by 12 amperes to yield 2 ohms as total resistance. In the product over sum method, the first pair is computed:

$$\frac{3 \text{ ohms} \times 6 \text{ ohms}}{3 \text{ ohms} + 6 \text{ ohms}} = \frac{18 \text{ ohms}}{9 \text{ ohms}} = 2 \text{ ohms}$$

The 2 ohms are then combined with the remaining 2 ohm branch.

$$\frac{2 \text{ ohms} \times 2 \text{ ohms}}{2 \text{ ohms} + 2 \text{ ohms}} = \frac{4 \text{ ohms}}{4 \text{ ohms}} = 1 \text{ ohms}$$
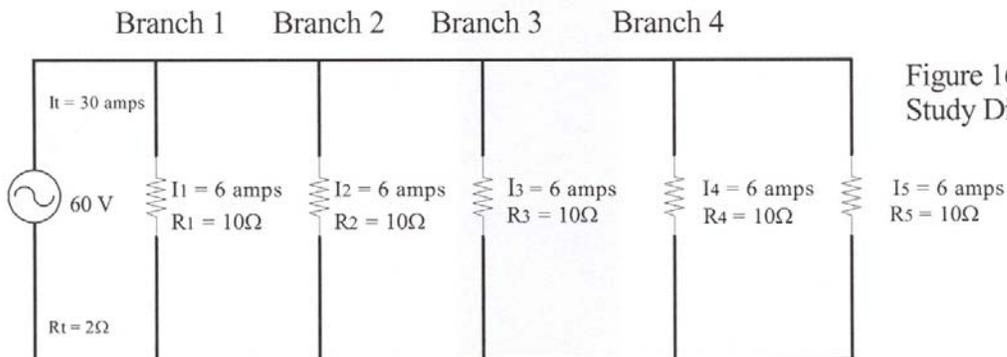

Figure 16
Study Diagram 16

Mini Quiz # 16 (see answers on last pages)

Using the above diagram, find total current and total resistance if all branch resistors were 15 ohms each. Underline the correct answer.

a. 3 amperes -- 20 ohms   b. 15 amperes -- 4 ohms
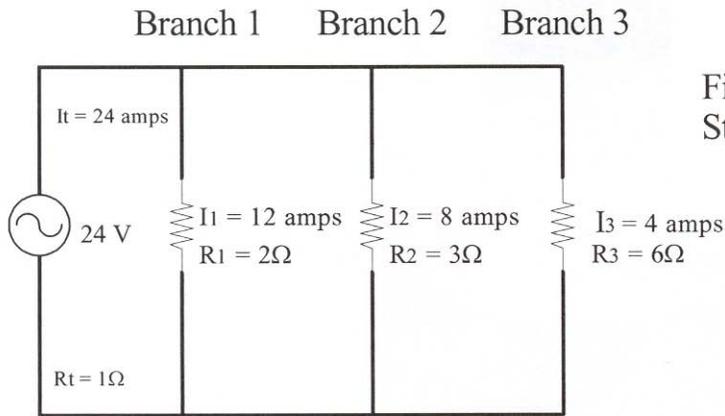c. 4 amperes -- 15 ohms   d. 20 amperes -- 3 ohms

Branch 1   Branch 2   Branch 3

Figure 17
Study Diagram 17

It = 24 amps

24 V

$I_1 = 12$ amps $I_2 = 8$ amps $I_3 = 4$ amps
$R_1 = 2\Omega$ $R_2 = 3\Omega$ $R_3 = 6\Omega$

Rt = 1Ω

Mini Quiz # 17 (see answers on last pages)

If $R_3 = 15$ ohms, $R_2 = 10$ ohms and $R_1 = 12$ ohms, what is the total resistance in the circuit. Underline the correct answer.

a. 2 ohms   b. 6 ohms
c. 4 ohms   d. 8 ohms

RULE #3

In the third method of finding the total resistance, the joint resistance of a parallel circuit is equal to the reciprocal of the sum of the reciprocals of the individual resistances or stated as a formula:

$$R_j = \cfrac{1}{\cfrac{1}{R_1} + \cfrac{1}{R_2} + \cfrac{1}{R_3}}$$

Substituting the values in Figure 17, this would be:

$$R_j = \cfrac{1}{\cfrac{1}{2} + \cfrac{1}{3} + \cfrac{1}{6}} \quad \text{or}$$

Converting the fractions to the common denominator 6

$$R_j = \cfrac{1}{\cfrac{3}{6} + \cfrac{2}{6} + \cfrac{1}{6}} = \cfrac{1}{\cfrac{6}{6}} = \cfrac{1}{1} \quad \text{or 1 ohm}$$

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This could also be converted to decimal form:

$$\frac{1}{.5 + .333 + .167} = \frac{1}{1} \text{ or 1 ohm}$$

All methods achieve the same solution.

PARALLEL CIRCUIT CHARACTERISTICS

The predominating characteristic of a series circuit is that it divides the <u>applied voltage</u> in direct proportion to circuit resistance.

In parallel circuits the predominating characteristic is that it divides current in direct proportion to circuit resistance. In control technology, these are very important assumptions. They serve to make even the most sophisticated solid state circuits understandable.

KIRCHHOFF'S LAW APPLIED TO PARALLEL CIRCUITS

Kirchhoff's Law for parallel circuits states that at any junction of conductors the algebraic sum of the currents is zero. This means that as many electrons leave the junction as enter it. Mathematically stated:

L_T (or total current) $\pm I_1 \ \pm I_2 \ \pm \ I_3 \ldots = 0$
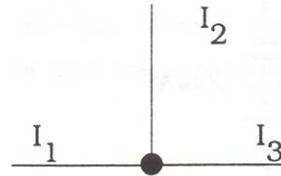
This merely presents a mathematical relationship which has been obvious from logical deduction. Further stated, it also applies to power in a parallel circuit. Total power is equal to the sum of the power consumed in the individual resistances.

SERIES-PARALLEL CIRCUITS

As the title indicates, series-parallel circuits are composite or compound circuits whose loads are connected in both series and parallel configurations. These circuits are extremely important to the refrigeration and air conditioning technician or application engineer because they are the basis for modulating or proportional controls.

These controls must deliver a variable response to a variable signal. The on-of f or snap switch control is incapable of delivering this response. Even the single pole double throw (SPDT) or floating control is ineffective for modulation when precise performance is required.

The bridge or Wheatstone bridge type circuits have the capabilities required for proportional control. Therefore, these controls predominate in the commercial-industrial applications where electrical control systems are utilized. Fundamentally, bridge circuits are a series-parallel circuit.

SERIES-PARALLEL CIRCUIT SOLUTIONS

Series-parallel circuits may be solved by application of the rules already given for simple series and simple parallel circuits. To do this, the series-parallel circuit is reduced to an equivalent, simplified circuit. Each group of parallel resistors is first replaced by it equivalent single resistance, and then the entire circuit is treated as a simple series circuit. This method is illustrated in Figures 18A, B, C and D.

In Figure 18A, the original circuit shows resistors R_3 and R_4 in series with each other. These resistors are in parallel with R_2. All of these resistors are in series with R_1.
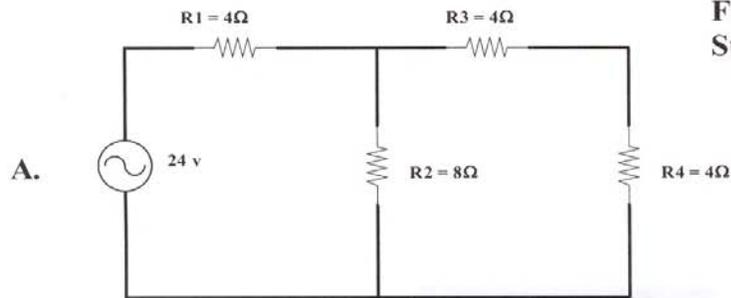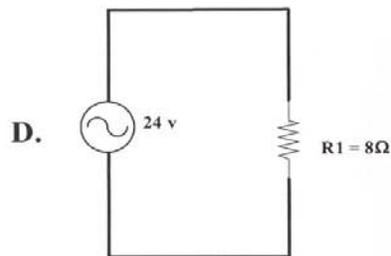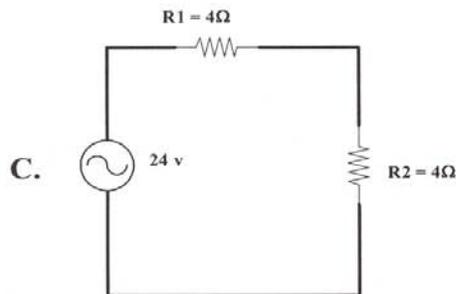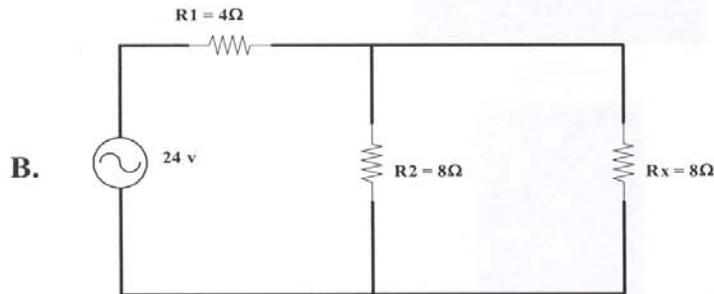

Figure 18
Study Diagram 18

A. R1 = 4Ω   R3 = 4Ω   24 v   R2 = 8Ω   R4 = 4Ω

B. R1 = 4Ω   24 v   R2 = 8Ω   Rx = 8Ω

C. R1 = 4Ω   24 v   R2 = 4Ω

D. 24 v   R1 = 8Ω

Mini Quiz # 18 (see answers on last pages)
If in 18A, R_2 is changed to 15 ohms and R_3 to 6 ohms with the other values unchanged, the total resistance would be. Underline the correct answer.

a. 8 ohms   b. 10 ohms
c. 11 ohms   d. 12 ohms

The recommended procedure is to start at the circuit's right side.

1. Resistor R_3 and R_4 are in series and additive so they are the equivalent of one 8 ohm resistor. Figure 18A is combined into 18B.

2. This results in an 8 ohm resistor in parallel with R_2 or another 8 ohm resistor (Figure 18B).

3. Reduce these two resistors down to an equivalent resistance. Hence rule #1 for combination circuits is applicable --- with equal resistors, the value for one resistor is divided by the number of branches.

$$\frac{\text{Resistance}}{\text{\# of branches}} = \frac{8 \text{ ohms}}{2 \text{ branches}} = 4 \text{ ohms (Figure 18C)}$$

4. Since R_1 and R_2 are in series and additive, the simple series equivalent now consists of one 8 ohm resistor as shown in Figure 18D.

Mini Quiz # 15 solution

$$\frac{1}{1/12 + 1/8 + 1/4 + 1/24} = \frac{1}{0.0833 + 0.125 + 0.25 + 0.04166} = \frac{1}{.5000} = \textbf{\textit{2 ohms}}$$

Mini Quiz # 16 solution

$$R_t = \frac{\text{Resistance of individual branch}}{\text{\# of Branches}} = \frac{15}{5} = \textbf{\textit{3Ω}}$$

and I_t = 60 volts/3Ω = ***20 amps***

Mini Quiz # 17 solution

Using the product over sum formula, find the equivalent value of resistors R_2 and R_3.

$$R_E = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6\ \Omega$$

Using the formula again for R_1

$$R_T = \frac{12 \times 6}{12 + 6} = \frac{72}{18} = \boldsymbol{4\ \Omega}$$

Mini Quiz # 18 solution

1. R_3 (6 ohms) + R_4 (4 ohms) = 10 ohms

2. $$R_E \frac{15 \times 10}{15 + 10} = \frac{150}{25} = 6\ \text{ohms}$$

3. R_T = 6 ohms + 4 ohms = 10 ohms