

TECH TIP # 50



One of a series of dealer contractor technical advisories prepared by HARDI wholesalers as a customer service.

A Crash Course in Fractions

Here is a quick refresher on working with fractions. This will be A very useful review if you haven't had to work with fractions for awhile.

Terms

Fractions -- One or more of the equal parts into which a unit may be divided. It is always written in the form of a division problem. As an example, one-eighth indicates one unit divided into eight equal parts and is written $1/8$, which would express the fact that it is equal to 1 divided by 8.

Numerator -- The number of equal parts within a fraction. For example, in the fraction $3/8$, the number 3 is the numerator of the fraction.

Denominator -- The number of equal parts into which the unit is divided. For example, in the fraction $3/8$, the number 8 is the denominator of the fraction.

Proper Fraction -- When the numerator of the fraction is less than the denominator. For example, $1/2$, $1/4$, $3/4$, etc., are proper fractions.

Improper Fractions -- The numerator of a fraction is equal to or greater than the denominator. For example, $8/3$, $4/4$, $20/15$, etc., are improper fractions.

Mixed Number -- A combination of a whole number and a fraction. For example: $5 \frac{1}{4}$ and $3 \frac{1}{2}$. Mixed numbers are sometimes typed for convenience as $5-1/4$ and $3-1/2$. Also, the mixed number such as $5 \frac{1}{4}$ means the same as $5 + \frac{1}{4}$.

Adding and subtracting fractions with the same denominator -- simply add or subtract numerators and use the common denominators.

Example: add

$$\frac{3}{16} + \frac{9}{16} + \frac{3}{16} = \frac{3+9+3}{16} = \frac{15}{16}$$

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Example: subtract

$$\frac{13}{16} - \frac{7}{16} = \frac{13-7}{16} = \frac{6}{16} = \frac{3}{8} \longleftarrow \text{Lowest terms}$$

Note: In the subtraction example, the fraction 6/16 was reduced to its lowest terms by dividing both the numerator and the denominator by 2 to obtain 3/8. Dividing or multiplying both the top and bottom of a fraction by the same number does not change the value of the fraction.

To add or subtract fractions with different denominators, you must first find a common denominator then add or subtract the resulting numerators.

Examples: add $\frac{3}{16} + \frac{5}{8} + \frac{5}{32}$

A common denominator is simply any one number that each of the original denominators will divide into evenly -- consider 64 ...

$$\begin{array}{ccc} & \times 4 & \\ \swarrow & & \searrow \\ 3 & & 12 \\ \hline 16 & \longrightarrow & 64 \end{array} = \frac{\quad}{64}$$

Divide 16 into 64 and obtain 4.
Then multiply 4 times 3 to obtain 12.
Repeat this procedure for each fraction to obtain all common (but still equivalent) fractions.

$$\frac{5}{8} = \frac{40}{64}$$

$$\frac{5}{32} = \frac{10}{64}$$

$$\frac{62}{64} = \frac{31}{32}$$

Note: we could have used 32 as the lowest or smallest common denominator and obtained the same result.

When a common denominator is not obvious, one can be determined by using the largest denominator in the group or multiplying two or more denominators together.

To change a whole number to a fraction, simply place it over 1 as a denominator.

Thus, 3 changed to a fraction would be 3/1.

To change a mixed number to a fraction, or more correctly, an improper fraction, multiply the whole number by the denominator and add the numerator. For instance: consider the mixed number 5-7/8:

$$5 \frac{7}{8} = \frac{(5 \times 8) + 7}{8} = \frac{40 + 7}{8} = \frac{47}{8}$$

To convert or simplify an improper fraction to a mixed number, divide the numerator by the denominator and if there is any remainder, it becomes the new numerator of the fraction along side the whole number. For example: consider 43/16

$$\frac{43}{16} = 16 \begin{array}{r} 2 \\ \overline{) 43} \\ \underline{-32} \\ 11 \end{array} = 2 \frac{11}{16}$$

If there is no remainder, the improper fraction simply reduces to a whole number, such as 32/8 reduces to 4.

The most common way to add or subtract mixed numbers is to consider the whole numbers and fractions --- adding or subtracting them accordingly --- then combining the results for the correct answer. Consider the need to add 2-1/4 and 1-3/16.

$$2 \frac{1}{4} = 2 + \frac{1}{4} = 2 + \frac{4}{16}$$

$$1 \frac{3}{16} = 1 + \frac{3}{16} = 1 + \frac{3}{16}$$

$$3 + \frac{7}{16}$$

16 is common denominator

Subtract 12-1/2 from 27-3/4 ...

$$27 \frac{3}{4} = 27 \frac{3}{4}$$

$$-12 \frac{1}{2} = -12 \frac{2}{4}$$

$$15 \frac{1}{2}$$

But consider this subtraction problem: $3\frac{7}{8}$ from $4\frac{1}{16}$...

$$\begin{array}{r} 4\frac{1}{16} = 4\frac{1}{16} \\ -3\frac{7}{8} = -3\frac{14}{16} \\ \hline \end{array}$$

After finding the common denominator for the fractions, note that we cannot subtract $14/16$ from the smaller value $1/16$. To complete this subtraction, we must “borrow” 1 from the whole number 4, reducing the whole number to 3, then add 1 or its fractional equivalent $16/16$ to $1/16$ to obtain $17/16$ which is larger than $14/16$.

$$\begin{array}{r} 4\frac{1}{16} = 3\frac{17}{16} \\ -3\frac{14}{16} = -3\frac{14}{16} \\ \hline \frac{3}{16} \end{array}$$

We can do this because.. $4\frac{1}{16} = 4 + \frac{1}{16} = 3 + 1 + \frac{1}{16} = 3 + \frac{16}{16} + \frac{1}{16} = 3 + \frac{17}{16}$

To multiply fractions, simply multiply the numerators together and place above the line and then multiply the denominators together to form a new denominator. It may then be possible to reduce the resulting fraction to its lowest form. For example: multiply

$$\frac{1}{2} \times \frac{3}{16} = \frac{1 \times 3}{2 \times 16} = \frac{3}{32}$$

Now multiply $5/9$ times $3/16$ $\frac{5}{9} \times \frac{3}{16} = \frac{5 \times 3}{9 \times 16} = \frac{15}{144}$

By dividing top and bottom by 3 we can reduce to simplest terms ...

$$\frac{15}{144} = \frac{5}{48}$$

It's also possible to multiply these two numbers by first exercising a *canceling* procedure

$$\frac{5}{9} \times \frac{3}{16} = \frac{5 \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times 16} = \frac{5}{48}$$

Note in the above approach, canceling was simply noting that 3 in the numerator would divide evenly into the 9 in the denominator to reduce the size of each number without changing the relationships. If 16 in the denominator had been 15, then we could have also cancelled 5 into 15

$$\frac{5}{9} \times \frac{3}{15} = \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{3}{\cancel{15}}} = \frac{1}{9}$$

To multiply mixed numbers, change the mixed numbers to fractions and then proceed as for fractions.

Example: $3\text{-}1/2$ times $4\text{-}3/4$.

Change to improper fractions... $3\frac{1}{2} = \frac{7}{2}$

$$4\frac{3}{4} = \frac{19}{4}$$

Then $\frac{7}{2} \times \frac{19}{4} = \frac{7 \times 19}{2 \times 4} = \frac{133}{8}$

Convert to a mixed number... $\frac{133}{8} = 16\frac{5}{8}$

To divide a fraction by a fraction, multiply the first fraction by the reciprocal (reverse) of the other.

For example: divide $11/9$ by $2/5$...

reversed $\frac{11}{9} \times \frac{5}{2} = \frac{55}{18} = 3\frac{1}{18}$

To divide mixed numbers, change the mixed numbers to improper fractions and proceed as detailed above. Example: $2\text{-}3/4$ divided by $1\text{-}5/8$

Change to improper fractions.....

$$2\frac{3}{4} = \frac{11}{4}$$

$$1\frac{5}{8} = \frac{13}{8}$$

$$\frac{11}{4} \text{ divided by } \frac{13}{8} = \frac{11}{4} \div \frac{13}{8} = \frac{11}{4} \times \frac{8}{13} = \frac{22}{13}$$

← reversed

↑ indicates division

$$\frac{22}{13} = 1\frac{9}{13}$$

Practice Problems

Change to improper fractions

1. $1\frac{1}{2}$ 2. $5\frac{2}{3}$ 3. $10\frac{25}{32}$ 4. $100\frac{3}{4}$ 5. $987\frac{57}{64}$

Reduce to lowest terms

6. $\frac{32}{64}$ 7. $\frac{27}{30}$ 8. $\frac{64}{32}$ 9. $\frac{100}{7}$ 10. $\frac{1000}{64}$

Add

11. $\frac{1}{16} + \frac{3}{16} + \frac{5}{16} + \frac{7}{16} =$ 12. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} =$
13. $\frac{2}{7} + \frac{3}{9} =$ 14. $1\frac{3}{8} + \frac{13}{15} + 5\frac{25}{32} =$
15. $\frac{3}{4} + \frac{5}{16} + 12\frac{1}{2} + \frac{5}{8} =$

Subtract

16. $\frac{3}{4} - \frac{1}{2} =$ 17. $\frac{5}{16} - \frac{9}{32} =$ 18. $1\frac{17}{32} - \frac{11}{27} =$
19. $7\frac{3}{16} - \frac{7}{16} =$ 20. $175\frac{2}{3} - 15\frac{27}{32} =$

Multiply

21. $\frac{1}{2} \times \frac{1}{2} =$

22. $\frac{3}{16} \times 4\frac{2}{3} =$

23. $3\frac{3}{4} \times 4\frac{4}{5} =$

24. $\frac{25}{32} \times 2\frac{7}{10} \times 12\frac{2}{3} =$

25. $5\frac{33}{200} \times 2\frac{1}{2} \times \frac{20}{1033} =$

Divide

26. $\frac{1}{2} \div \frac{1}{2} =$

27. $4 \div \frac{4}{5} =$

28. $\frac{3}{16} \div 6 =$

29. $11\frac{21}{33} \div 10\frac{12}{15} =$

30. $100 \div \frac{1}{10} =$

Answers

1. $\frac{3}{2}$

2. $\frac{17}{3}$

3. $\frac{345}{32}$

4. $\frac{403}{4}$

5. $\frac{63225}{64}$

6. $\frac{1}{2}$

7. $\frac{9}{10}$

8. 2

9. $14\frac{2}{7}$

10. $\frac{125}{8} = 15\frac{5}{8}$

11. $\frac{16}{16} = 1$

12. $\frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$

13. $\frac{18}{63} + \frac{21}{63} = \frac{39}{63} = \frac{13}{21}$

14. $1\frac{180}{480} + \frac{416}{480} + 5\frac{375}{480} = 6\frac{971}{480} = 8\frac{11}{480}$

15. $\frac{12}{16} + \frac{5}{16} + 12\frac{8}{16} + \frac{10}{16} = 12\frac{35}{16} = 14\frac{3}{16}$

16. $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

17. $\frac{10}{32} - \frac{9}{32} = \frac{1}{32}$

18. $1\frac{459}{864} - \frac{352}{864} = 1\frac{107}{864}$

19. $6\frac{19}{16} - \frac{7}{16} = 6\frac{12}{16} = 6\frac{3}{4}$

20. $175\frac{64}{96} = 174\frac{160}{96} - 15\frac{81}{96} = 159\frac{79}{96}$

$$21. \frac{1}{4}$$

$$22. \frac{\cancel{3}}{16} \times \frac{7}{\cancel{14}} = \frac{7}{8}$$

$$23. \frac{\cancel{15}}{4} \times \frac{6}{\cancel{5}} = \frac{18}{1} = 18$$

$$24. \frac{\cancel{25}}{32} \times \frac{9}{\cancel{10}} \times \frac{19}{\cancel{38}} = \frac{855}{32} = 26 \frac{23}{32}$$

$$25. \frac{\cancel{1033}}{200} \times \frac{1}{\cancel{5}} \times \frac{1}{\cancel{20}} = \frac{1}{4}$$

$$26. \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{1} = \frac{1}{1} = 1$$

$$27. \frac{4}{1} \times \frac{5}{4} = \frac{5}{1} = 5$$

$$28. \frac{\cancel{3}}{16} \times \frac{1}{\cancel{6}} = \frac{1}{32}$$

$$29. \frac{\cancel{384}}{33} \times \frac{5}{\cancel{15}} = \frac{960}{891} = 1 \frac{69}{891}$$

$$30. \frac{100}{1} \times \frac{10}{1} = 1000$$